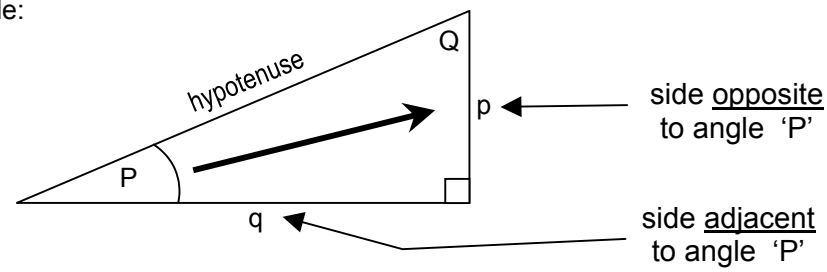
Trigonometry - Activity 9

Tangent ratio: Introduction.

Consider this right angled triangle:



Side 'p' is opposite to angle 'P', and side 'q' is adjacent to angle 'P'. ('Adjacent' means 'beside').

In right angled triangles, the term 'adjacent' is never used to refer to the longest side opposite to the right angle. This is always called the 'hypotenuse'. Only the shorter sides, the 'legs' of the right triangle, can be 'adjacent' sides.

1) What side is opposite angle 'Q' ? _____

2) What side is adjacent to angle 'Q' ? _____

For an angle less than 90º in a right angled triangle, the following ratio: $\frac{\text{opposite}}{\text{adjacent}}$ is called the 'tangent ratio' of the angle.

For example, the tangent ratio of angle 'P' above can be found as follows:

'tangent ratio of P' is written as: 'tan P'

$$\tan P = \frac{\text{opposite}}{\text{adjacent}} = \frac{p}{q}$$

3) Complete the following:

$$\textbf{tan}\ Q =$$

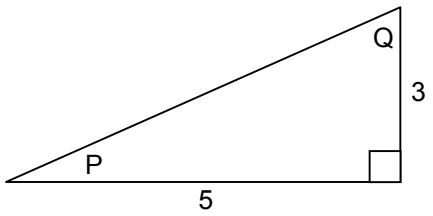
4) Start Maths Helper Plus and load the 'R2 - Tangent ratio 1.mhp' document.
This document calculates tangent ratios for right triangles.

5) Press the F5 key to **display the parameters box**:
These edit boxes: 'A', 'B' and 'C' are used to set up the right triangle as follows:

If 'C' = 1, then the vertical leg will be 'A' units long, and the horizontal leg will be 'B' units long.
If 'X' = 1, then the tangent ratio is calculated for angle 'P'. If 'X' = 2, then the calculations are based on angle 'Q'.

To change a value on the parameters box: (1) **click** in the centre of its edit box, (2) **press** the backspace key to delete the old value, (3) **type** the new value and then (4) **click** the 'Update' button.

6) Complete the tangent ratio calculations for the two marked angles in the right angled triangle below:



(a) $\tan P = \dfrac{\text{opposite}}{\text{adjacent}}$

$= \dfrac{3}{5}$

$=$ _____

(b) $\tan Q = \dfrac{\text{opposite}}{\text{adjacent}}$

$=$ _____

$=$

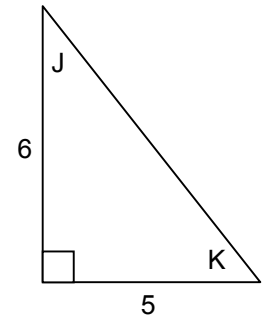
7) Use Maths Helper Plus to **check and correct your work**.

- Enter the parameters box values to create the triangles in question 6:
  Part (a): A = 3, B = 5, C = 1, X = 1 Part (b): A = 3, B = 5, C = 1, X = 2
- Compare the calculations displayed by the computer with yours and correct any mistakes.

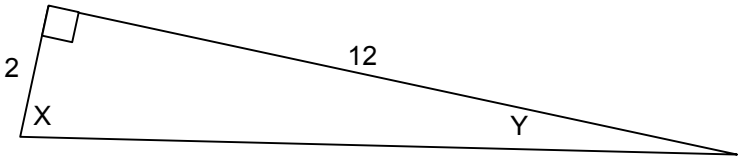
Hint: Maths Helper Plus always draws the triangle in the same orientation and with the angles named 'P' and 'Q'. This doesn't matter. The important thing is to enter the opposite and adjacent sides correctly. If 'X' = 1 on the parameters box, then 'A' is the opposite side and 'B' is the adjacent side. If 'X' = 2, then 'B' is the opposite side and 'A' is the adjacent side.

8) Calculate the tangent ratios for each of the two marked angles in the triangles below.
Show all working, and use Maths Helper Plus to help you check and correct your work.

(a)



(b)



Investigation: If the size of a triangle is changed, what effect will this have on the tangent ratios?

In this activity, you will use Maths Helper Plus to experiment with right angled triangles while recording some results in a table. You will then be able to answer this question and give a reason for your answer.

9) Set up the parameters box as follows: A = 3, B = 4, C = 1, X = 1
The triangle displayed in Maths Helper Plus now has angle 'P' = 36.8699º, with the opposite side = 3, adjacent side = 4, and the calculated value of tan 36.8699º = 0.75.

10) Change the size of the triangle in small steps, recording the side lengths and 'tan P' values each time.

(a) Click on the 'C' edit box on the parameters box.
(b) Click on the 'slider' button. (On the right-hand side of the parameters box.)
(c) Quickly press and release the keyboard up arrow key to increase the triangle size.
(d) Record the opposite and adjacent side lengths, and 'tan P' in the table below.

Repeat these steps until the table is full.

angle P	opposite side	adjacent side	tan P
36.8699º	3	4	0.75

11) Does the size of a triangle have any effect on the tangent ratios of its angles ? _____

Explain your answer:

12) To **test your findings from question 11 for some other triangles**, change the 'A' and 'C' values in the parameters box to create other tangent ratios, then vary the 'C' value as in '10' above.

Name:_____ Class:_____

Trigonometry - Activity 10

Tangent ratio: Finding unknown angles in right triangle.

The tangent ratio of an angle θ in a right angled triangle is defined as: $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

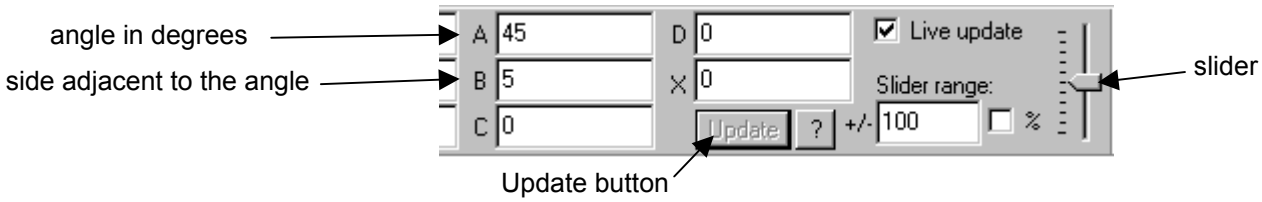
The value of 'tan θ' is found to depend only on the size of the angle, 'θ', so that 'tan 30º' will be the same for all right angled triangles having a 30º angle.

You will now create a table of tangent ratios that can be used to find unknown sides and angles in many different triangles.

1) Start Maths Helper Plus and load the 'R2 - Tangent ratio 2.mhp' document.
This document calculates tangent ratios for angles in right angled triangles.

2) Press the F5 key to **display the parameters box**:

Edit boxes: 'A' and 'B' are used as follows:



3) To **calculate the tangent ratio for any angle**,

- click on the centre of the edit box for 'A' on the parameters box.
- press backspace to delete the existing angle.
- type the new angle.
- click the 'Update' button.

Use Maths Helper Plus to calculate these tangent ratios:

(a) tan 15º = _____ (b) tan 30º = _____ (c) tan 45º = _____ (d) tan 60º = _____

4) Creating a **table of tangent ratios**

A table of tangent ratios can be used to find unknown angles and sides in right angled triangles.
Use Maths Helper Plus to calculate the tangent ratio for the angles in the table below. Write the values in the table. (In the table, the angle is called 'A'.)

Hint: To quickly change the angle on the parameters box, first click on the 'A' edit box, backspace and change the number to '5', then click on the slider. Now use the up and down keyboard arrows to change the angle.

Aº	tan Aº	Aº	tan Aº	Aº	tan Aº
5		35		65	
10		40		70	
15		45		75	
20		50		80	
25		55		85	
30		60		90	

5) Maths Helper Plus tries to calculate **tan 90º**, but doesn't really give the true answer.
What is the value of tan 90º ? _____ [**Hint**: When 'A' = 90º, how long is the opposite side?]

6) Use the table from question 4 to **find the unknown angles** below:

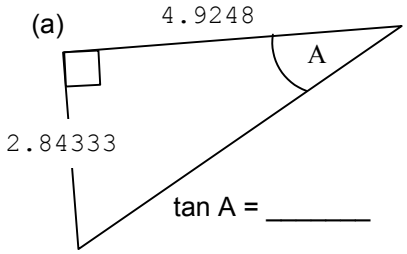
(a) tan A = 0.2679 (b) tan B = 11.4301 (c) tan C = 1 (d) tan D = 1.7321 (e) tan E = 0.3640

A = _____ B = _____ C = _____ D = _____ E = _____

7) Find the unknown angles in the right angled triangles below.

Use the two sides given to calculate the tangent ratio for the unknown angle, then use the table of tangent ratios that you created in question 4 to find the angle.

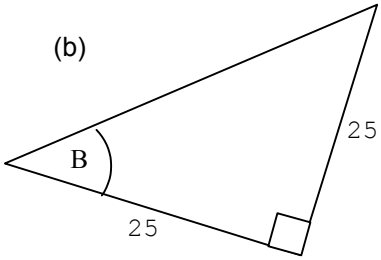
(a)



tan A = _____

so A = _____

(b)



tan B = _____

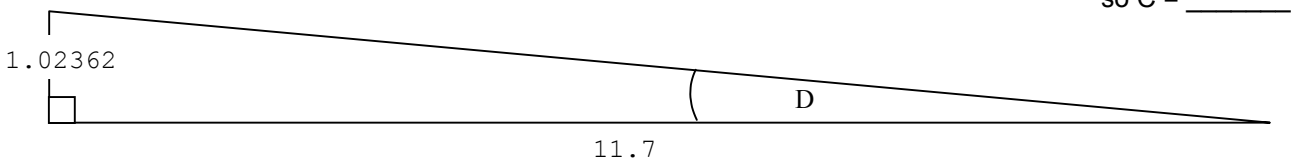
so B = _____

(c)

tan C = _____

so C = _____

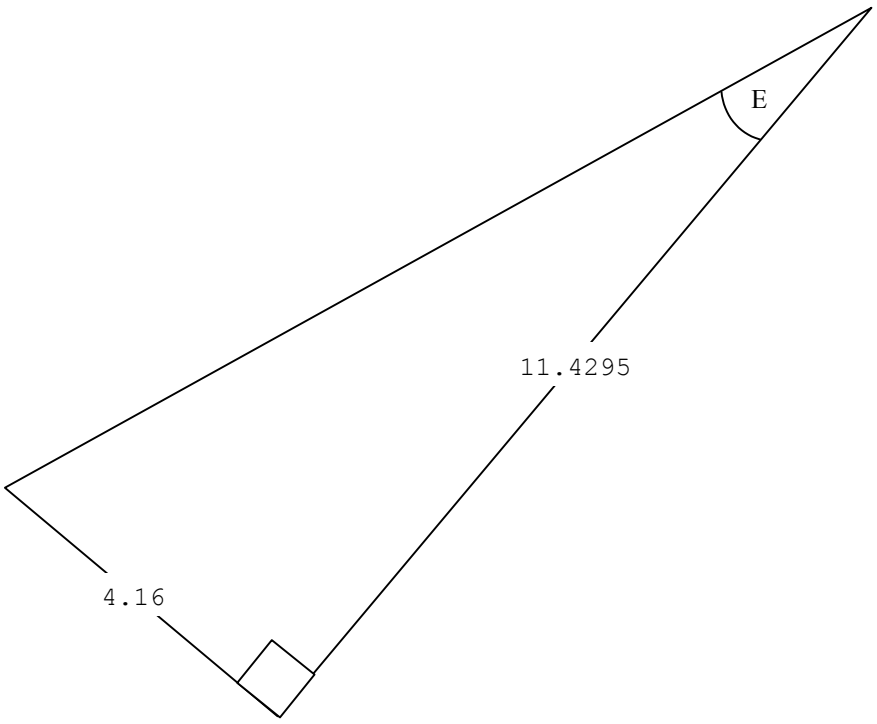
(d)



tan D = _____

so D = _____

(e)



tan E = _____

so E = _____

Trigonometry - Activity 11

Tangent ratio: Finding unknown opposite or adjacent sides in right triangles.

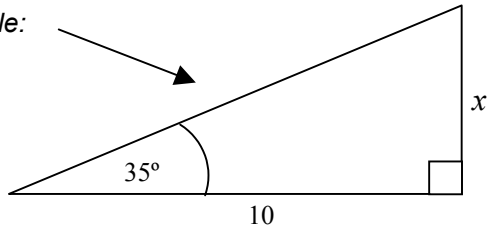
In this activity you will practice finding unknown sides of right triangles using the tangent ratio, then use Maths Helper Plus to correct your working and answers.

The tangent ratio can be used to calculate the length of one of the shorter sides (legs) of a right triangle. To do this you need the tangent ratio for one of the smaller angles, and the length of one leg of the triangle.

There are two types of these problems, depending on whether you are finding the 'opposite' or 'adjacent' side.

Problem type 1: Finding the opposite side.

Example 1: Use the tangent ratio to find the unknown side in this triangle:



Solution:

$$\tan 35° = \frac{opposite}{adjacent} = \frac{x}{10}$$

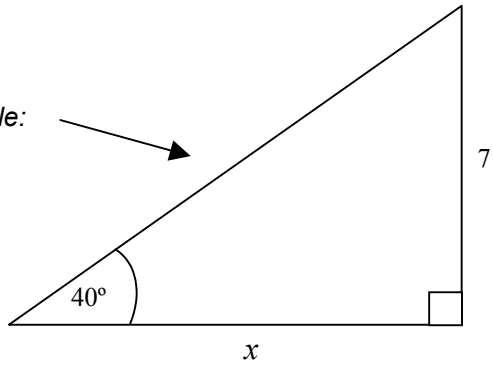
$$so \quad x = 10 \times \tan 35°$$

But $\tan 35º = 0.700208$ (From a table of tangent ratios, or scientific calculator with a 'tan' button.)

$$\text{so } x = 10 \times 0.700208$$
$$= 7.00208$$

Problem type 2: Finding the adjacent side.

Example 2: Use the tangent ratio to find the unknown side in this triangle:



Solution:

$$\tan 40° = \frac{opposite}{adjacent} = \frac{7}{x}$$

$$so \quad \tan 40° \times x = 7$$

$$and \qquad x = \frac{7}{\tan 40°}$$

But $\tan 40º = 0.8391$ (From a table of tangent ratios, or scientific calculator with a 'tan' button.)

$$so \quad x = \frac{7}{0.8391}$$
$$= 8.34228$$

1) Start Maths Helper Plus and load the 'R2 - Tangent ratio 3.mhp' document.
This document solves for unknown sides in right triangles using the 'tan' ratio.

2) Press the F5 key to **display the parameters box**. (See below.) 'A', 'B' and 'C' have meanings as shown below:

'A', one of the two smaller angles of the triangle:

The **adjacent** side to angle 'A':

The **opposite** side to angle 'A':

Only one of the legs of the triangle can be entered at a time. Set the unknown side length to zero.
To enter a value, click on an edit box, backspace to clear the old value, type the new value then click 'Update'.
The diagram will always draw the adjacent side horizontally, and the opposite side vertically.

NOTE: **If the diagram becomes too big** for your computer screen, press the 'F10' key to make it smaller.
To make the diagram bigger, hold down 'Shift' while you press 'F10'.

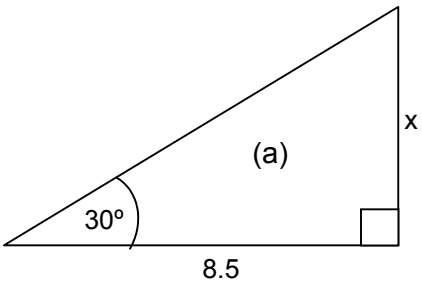
3) Use the tangent ratio to **find the unknown leg** in each of the triangles below. For each triangle:

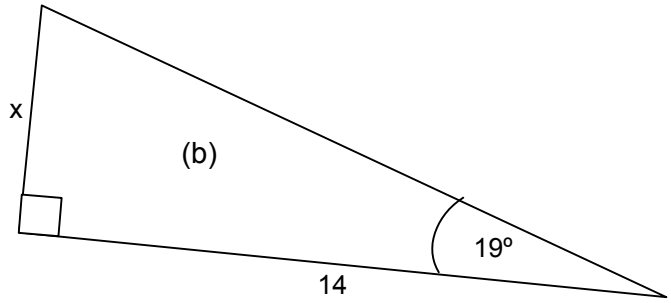
Calculate → Check → Correct

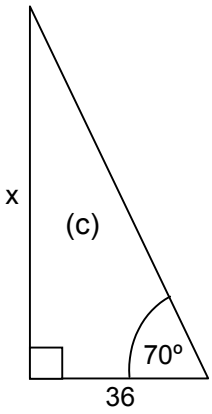
1. Identify the opposite and adjacent legs for the angle given.
2. Obtain the tangent ratio of the angle from a calculator or printed table.
3. Write the tangent ratio rule and substitute the known values.
4. Calculate the answer, showing all working steps.

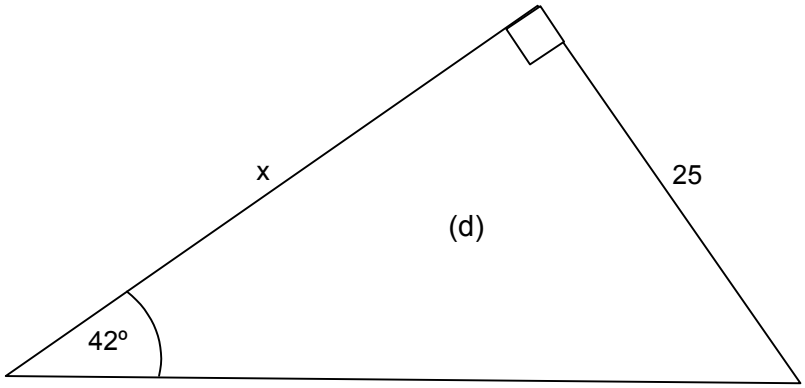
1. Enter the angle and given side into the Maths Helper Plus edit boxes.
2. Set the unknown side to zero.

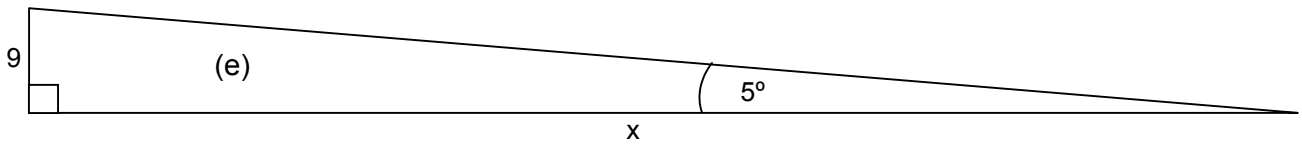
1. Compare your working steps and answers with Maths Helper Plus.
2. Fix your mistakes.

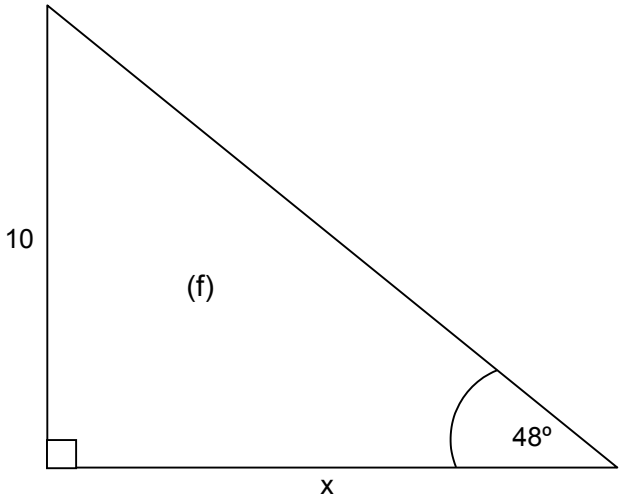








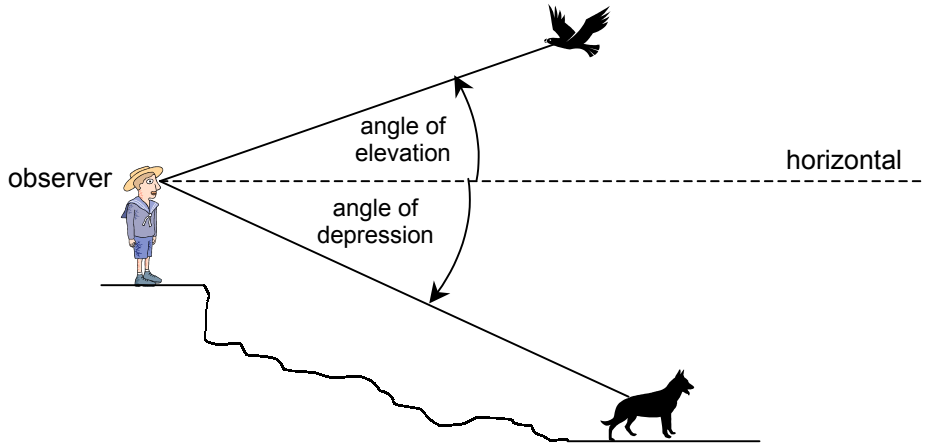




Trigonometry - Activity 12

Tangent ratio: Angles of elevation and depression.

Angles of elevation and depression are measured from the horizontal, like this:



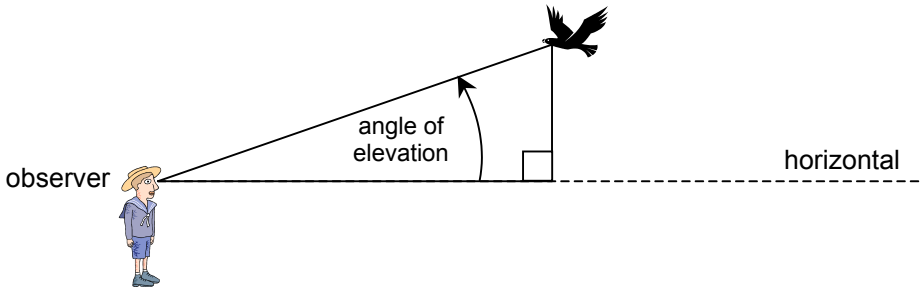
For an object that is higher than the observer, the angle of elevation is measured upwards from the horizontal to the straight line between the observer and the object.

For an object that is lower than the observer, the angle of depression is measured downwards from the horizontal to the straight line between the observer and the object.

An angle of elevation and depression can help us find an unknown distance. All we need to do is construct a right triangle containing the angle and then use the tan ratio.

Case 1 - Angles of elevation

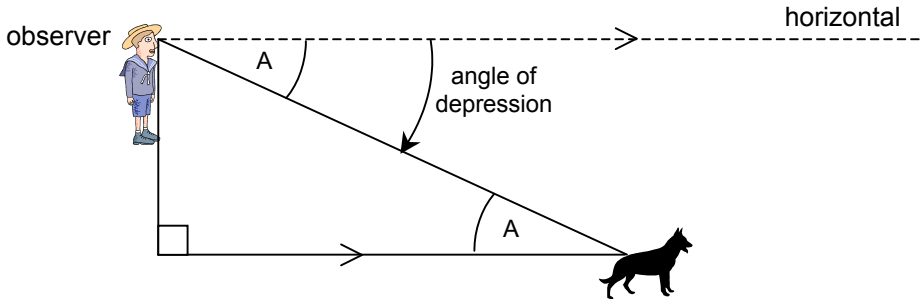
Given an angle of elevation and a horizontal or vertical distance between the object and the observer, we can construct a right angled triangle like this:



Now we can use the tan ratio to find the unknown horizontal or vertical side.

Case 2 - Angles of depression

Given an angle of depression and a horizontal or vertical distance between the object and the observer, we can construct a right angled triangle like this:



The angle of depression, 'A', is the same as the angle in the triangle (near the dog) because the base of the triangle is parallel to the horizontal.

Once again, the tan ratio can be used to find the unknown horizontal or vertical side of this triangle.

1) Start Maths Helper Plus and load the 'R2 - Tangent ratio 4.mhp' document.
This document solves 'angle of elevation' and 'angle of depression' problems.

2) Press the F5 key to **display the parameters box**. (See below.) 'A', 'B', 'C' and 'X' have meanings as shown:

3) For each of the word problems below:

1. Draw a labelled diagram of the situation similar to those on the front of this sheet. Include the horizontal line of sight from the observer and a right angled triangle.
2. Use the tangent ratio to calculate the unknown leg of the right triangle. Show all working.
3. Make sure you have answered the original question. Include units if necessary.
4. Use Maths Helper Plus to correct your diagram, working steps and answer.

NOTE: **If the diagram becomes too big** for your computer screen, press the 'F10' key to make it smaller. To make the diagram bigger, hold down 'Shift' while you press 'F10'.

a) A ladder placed on a flat horizontal surface rests against a vertical wall with an angle of elevation of 65º. The foot of the latter is 2 metres from the base of the wall. Find the hight of the point where the ladder touches the wall.

To check with Maths Helper Plus, set: A=65, B=2, C=0, X=1

b) A hungry cat spies a bird in a tree. The bird is 3.4 metres above the ground, and the angle of elevation to the bird from the cat's point of view is 40º. How far is the cat from the base of the tree ?

To check with Maths Helper Plus, set: A=40, B=0, C=3.4, X=1

c) The common archer fish (Toxotes chatareus) is able to squirt a water jet from its mouth with deadly accuracy to shoot down insects crawling on leaves and stems of overhanging vegetation. An archerfish at the surface of the water notices a dragonfly resting on a twig at an angle of elevation 77º. The dragonfly is exactly above a point on the water that is 30cm from the fish. How high is the dragonfly above the water surface ?

To check with Maths Helper Plus, set: A=77, B=30, C=0, X=1

d) A parachutist has an altitude of 320 metres when directly above a high voltage power pole. She spies the landing target and estimates that the angle of depression to the target is 33º. What is the approximate horizontal distance from the power pole to the target ?

To check with Maths Helper Plus, set: A=33, B=0, C=320, X=2

e) A fishing boat is 120 metres away from a point on the water that is directly above a school of large fish. The sonar fish detector on the boat indicates that the angle of depression from the boat to the fish is 55º. How deep are the fish ?

To check with Maths Helper Plus, set: A=55, B=120, C=0, X=2